



Interpreting three-dimensional structural topology optimization results

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Abstract

Topology optimization result is usually a gray level image in discrete finite elements, which is hard to interpret from a design point of view. It is especially difficult to interpret three-dimensional topology optimization result. Although there are many techniques for interpreting topology optimization results, most of them only deal with two-dimensional problems, and some of the interpreting techniques must be performed manually. This paper presents an automated process for interpreting three-dimensional topology optimization result into a smooth CAD representation. A tuning process is employed before the interpretation process to improve the quality of the topology optimization result. In the tuning process, a continuity analysis is first used to transform the porous topology formed by the elements with intermediate densities in the topology optimization result into clear 0–1 elements. Next, trivial solids and trivial voids, which are useless in mechanics but will complicate the implementation of the interpretation process, are filtered out. After the tuning process, the three-dimensional structure is divided into representative cross-sections. On each cross-section, a density redistribution algorithm transfers the black-and-white finite element topology optimization result into a smooth density contour represented by B-spline curves on each cross-section. A three-dimensional CAD model is obtained by sweeping through these cross-sections.

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1. Introduction

Topology optimization is the technique that finds the optimal layout of the structure within a specified design domain. The possible constraints in the topology optimi-

zation problems are the usable volume or material, and design restrictions (or geometry constraints, such as holes or non-design regions) in the design domain, under certain applied loads and boundary conditions.

Bendsøe and Kikuchi presented the homogenization method for topology optimization in 1988 [1], which is a milestone for topology optimization. The homogenization method was based on the assumption of a micro-structure in which the properties are homogenized. Two other approaches were proposed in the early

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stressed elements to achieve the optimal design. Rozvany provided a complete survey on topology optimization [17]. The density distribution method seems to be the most convenient method and is used here to obtain topology optimization result.

After the important advance by Bendsoe and Kikuchi in the field of topology optimization, many researchers paid attention to integrating structural topology optimization and shape optimization since early 1990s. Some researchers treat structural design optimization as a three-phase design process, and aim at developing an automated design process [4,5,10]. Phase I is the topology generation process, in which the optimal topology of the structure is generated. In the Phase II topology interpretation process, various approaches are used to interpret the topology optimization result. Phase III is the detailed design phase, in which the shape and size optimization are implemented.

The approaches used in the Phase II interpretation process can be roughly divided into three categories: image interpretation approach [15,2,13,4,5,10], density contour approach [9,22,8], and geometric reconstruction approach [19]. The image interpretation approach uses graphic facilities or computer vision technologies to represent the boundary of the black-and-white finite element topology optimization result. The density contour approach generates the boundaries of the structure by redistributing densities from the topology optimization result. In the geometric reconstruction

approach, the boundaries are represented by the mathematical geometric reconstruction technique.

Topology optimization result is usually a gray level image in discrete finite elements, which is hard to interpret from a design point of view. It is especially difficult to interpret three-dimensional topology optimization result. Although there are many interpretation techniques available in the literature, most of them only deal with the two-dimensional problems. For example, the image interpretation approach commonly seen in the literature can only represent the boundaries of the two-dimensional gray level topology result. Moreover, some of the interpreting techniques must be performed manually. Thus, a technique that can handle both two- and three-dimensional

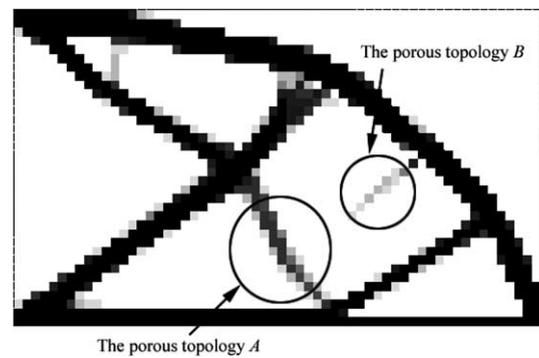


Fig. 2. The illustration of the porous topology.

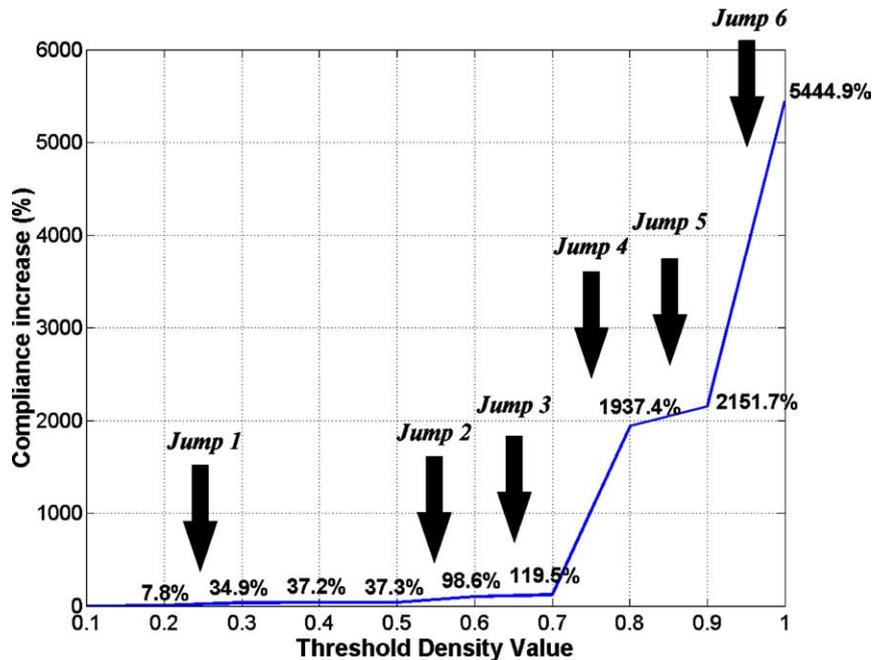


Fig. 3. The compliances of the cantilever beam example using different threshold densities.

problems and can be performed without human intervention is desired.

The concept of using density contour to interpret the topology optimization results was first introduced by Kumar and Gossard in 1996. In the density contour approach, the boundaries of topology optimization result are represented by nodal density contours, which are computed by material densities. Unlike the image interpretation approach, the density contour approach has the potential of expanding to three-dimensional structural design.

This paper presents an automated process for interpreting three-dimensional topology optimization results from the finite element representation into a smooth CAD model that can be used in later design stages. As shown in Fig. 1, the density distribution method, or the so-called simple isotropic material with penalization (SIMP) method, is used in the topology optimization. A tuning process is employed before the interpretation

process to improve the quality of the topology optimization result. In the tuning process, a continuity analysis is first used to transform the porous topology formed by the elements with intermediate densities in the topology optimization result into clear 0–1 elements. Referring to Fig. 1, the normalized material densities ($\rho_{i,final}$) are obtained from topology optimization. In the continuity analysis, a density value $\rho_{continuous}$ is to be decided so that a continuous topology can be obtained if only the elements whose normalized material densities are equal to or greater than $\rho_{continuous}$ are considered. The normalized material densities of these elements are set to be 1. On the other hand, the normalized material densities which are less than $\rho_{continuous}$ are set to be 0. Next, trivial solids and trivial voids, which are useless in mechanics but will complicate the implementation of the interpretation process, are filtered out.

After the tuning process, the three-dimensional structure is divided into representative cross-sections. A den-

Table 1
The topology of different threshold density

$\rho_{continuous}$	Topology	$\rho_{continuous}$	Topology
0.1		0.6	
0.2		0.7	
0.3		0.8	
0.4		0.9	
0.5		1.0	

sity redistribution algorithm then evaluates the nodal density values from the element normalized material densities of elements on each cross-section. Spline curves are used to generate the boundary curves using the boundary nodes as control points on each cross-section. Till this step, the whole procedure is fully automated and is applicable to both two- and three-dimensional structural design. A three-dimensional CAD model is obtained by sweeping through the cross-sections in any CAD software. In this paper, two-dimensional

examples are first used to describe this process, and then the process is directly extended to three-dimensional structures.

2. The continuity analysis of the topology optimization result

Topology optimization with penalty function attempts to generate the result with elements whose

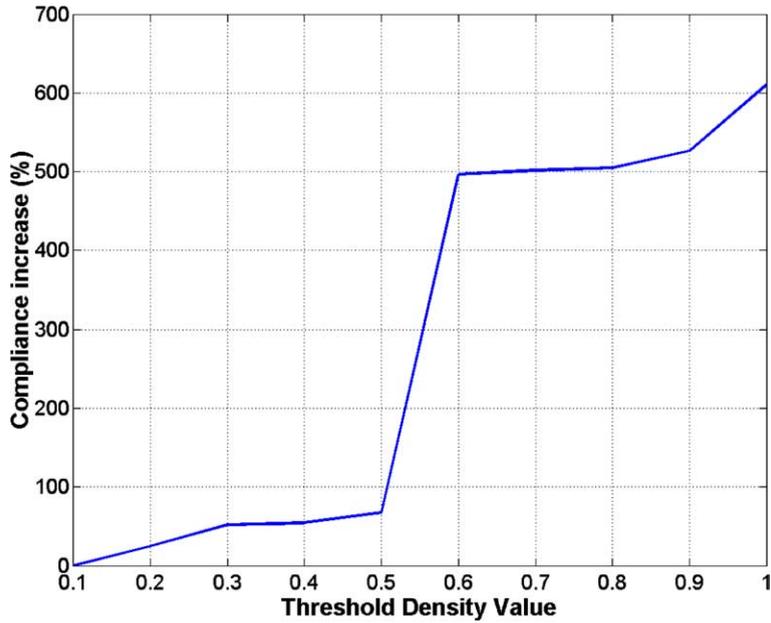


Fig. 4. The compliances of the suspension arm example using different threshold densities.

Table 2
The topology of different threshold density of suspension arm example

$\rho_{\text{continuous}}$	Topology	$\rho_{\text{continuous}}$	Topology
0.1		0.6	
0.2		0.7	
0.3		0.8	
0.4		0.9	
0.5		1.0	

normalized material densities are either 0 or 1. But the “porous topology” [16], which is formed by elements with intermediate normalized material densities between 0 and 1, often occurs practically.

Practically the most convenient way is to force the intermediate normalized material densities which are equal to or greater than a given threshold value of 1 directly. On the other hand, the densities that are less than the given threshold value are forced to be 0. The continuity of the structure should be of the most important consideration in deciding this threshold value. As shown in Fig. 2, if the threshold value is high and a portion of porous topology *A* is discarded, a discontinuous structure will be generated, which may greatly affect the compliance of the structure. On the other hand, the continuity of the structure is not affected if the porous topology *B* is filtered out.

A structure with continuous topology is stiffer than the discontinuous one, in other words, the compliance of the structure with continuous topology is lower. Thus, a proper threshold density value $\rho_{\text{continuous}}$ has to be decided by comparing the compliances of the structure using different threshold density values.

Fig. 3 shows the compliances of the cantilever beam example using different threshold density values. The *x*-axis is the threshold density value starting from 0.1 to 1.0, with increment of 0.1. Using the compliance when the threshold density is 0.1 as the base, the increasing percentages of the compliances are plotted. The respective topology results are shown in Table 1.

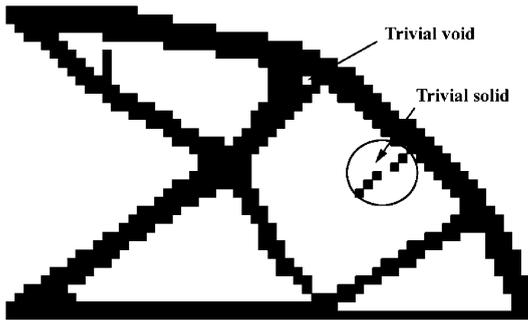


Fig. 5. The trivial solids and void in cantilever beam example.

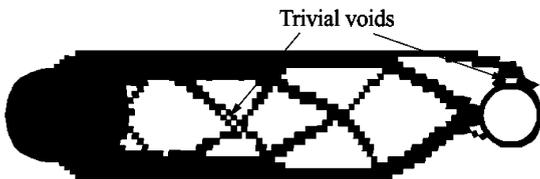


Fig. 6. The trivial voids in the suspension arm example.

As shown in Fig. 3, there are six “jumps” in compliance when the threshold density value increases. Compared with the figures in Table 1, there is more than 60% increase (jump 2) in compliance upon increasing when the threshold density value from 0.5 to 0.6, because a discontinuity of the topology occurs (indicated by a circle). Other discontinuities result in “jumps” in the compliance value. In this research, if the increase in compliance is more than 10% at certain threshold density value, the threshold density value is set to be $\rho_{\text{continuous}}$. In the cantilever beam example, $\rho_{\text{continuous}}$ is decided to be 0.2.

Fig. 4 shows the compliances of the suspension arm example using different threshold density values. The respective topology results are shown in Table 2. As shown in Fig. 4, there is a 24% increase in compliance when the threshold density value increases from 0.1 to 0.2, because a discontinuity of the topology occurs. Therefore, $\rho_{\text{continuous}}$ is decided to be 0.1.



Fig. 7. The result of the cantilever beam example after the trivial solid filter.



Fig. 8. The result of suspension arm example after the trivial void filter.

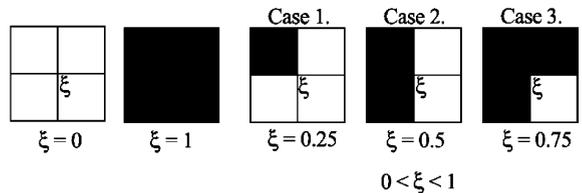


Fig. 9. The three types of nodal density value.

3. The trivial solid and void filter

After the continuity analysis, it is observed that trivial solids and voids appear in the topology result. Fig. 5 shows the typical trivial solids, such as islands or small salient, in the cantilever beam example. These trivial solids are useless in mechanics and will complicate the implementation of the interpretation process. Thus, the trivial solids should be filtered out before implementing the interpretation process.

Trivial voids are the voids containing few elements whose normalized material densities are 0. Figs. 5 and 6 show the trivial voids in the examples of cantilever beam and suspension arm. Similar to trivial solids, trivial voids are useless in mechanics, as it complicates the interpretation of topology optimization result, which should be filtered out.

In this research, a trivial solid and void filter is developed to filter those trivial topologies. The trivial solid is defined as the solid containing a group of elements that do not connect with other groups, or connect with other groups in only one node. Fig. 7 is the result of cantilever beam example after applying the trivial solid filter.

There can be many voids in the topology optimization result. In this research, if the number of elements contained in a void is less than 1% of the total number of elements multiplied by ρ_0 , the ratio of the amount of usable material, it is defined as a trivial void. Fig. 8 shows the topology result of the suspension arm exam-

ple after the trivial void filter. The checkerboard like trivial voids shown in Fig. 6 are filtered out.

4. The density contour approach

We hope to transfer the topology optimization result into a smooth CAD model that can be used in later design stages. As discussed earlier, Kumar and Gossard [9], Youn and Park [22] and Hsu et al. [8] use the density contour approach to interpret the topology optimization result to obtain a smooth boundary. In their works, the element densities of topology optimization result are redistributed into nodal densities, and the nodal density contour is generated at a specified density value. This contour is then transferred into a smooth CAD representation (such as B-splines) of the structure. This research also adopts the density contour approach, as discussed below.

Youn and Park [22] evaluate the nodal density (ξ) by averaging the normalized material densities of the neighboring elements as follows:

$$\xi_k = \frac{1}{W} = \sum_{e=1}^W \rho_{k,e} \tag{1}$$

where ξ_k is the density of the k th node, W is the number of neighboring elements at this node, and $\rho_{k,e}$ is the normalized material density of the e th neighboring element of the k th node. In two-dimensional topology

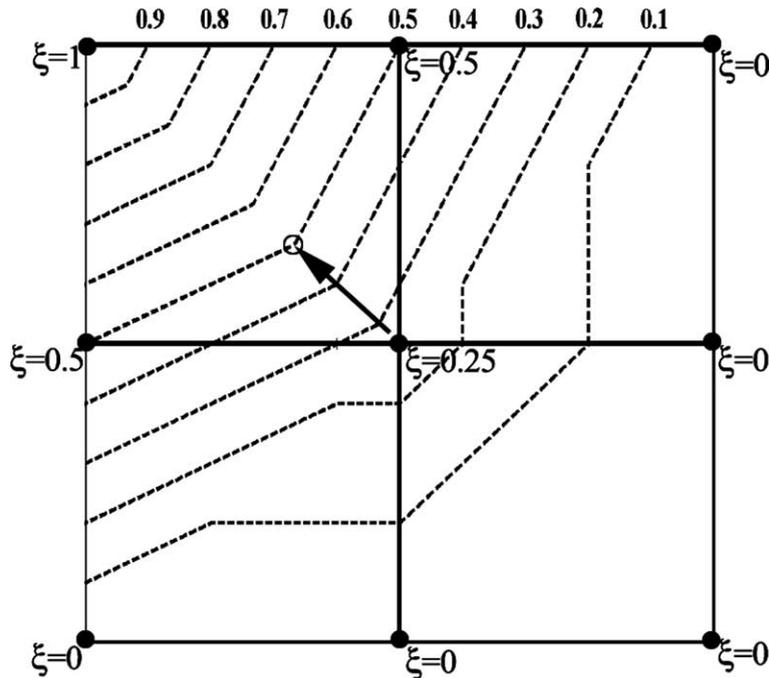


Fig. 10. The nodal densities and nodal density contour of Case 1.

optimization, almost all nodes have four neighboring elements, and thus $W = 4$.

Fig. 9 shows the three possibilities of nodal density values using Eq. (1), $\xi = 0$, $\xi = 1$ and $0 < \xi < 1$. If the node is inside a void, the normalized material densities of all neighboring elements are 0 and $\xi = 0$. On the other hand, if the node is inside a solid, then $\xi = 1$. For the nodes on the boundary between solid and void, the nodal density value is between 0 and 1. Fig. 9 also shows three possible cases, $\xi = 0.25$, $\xi = 0.5$ and $\xi = 0.75$.

Fig. 10 shows the nodal densities and nodal density contours of Case 1 in Fig. 9. It is clear that the contour for $\xi = 0.5$ properly describes the boundary of the solid. The nodal density contours are linearly interpolated from nodal densities. Spline curves, which are commonly used in CAD models, are generated to approximate the nodal density contour for $\xi = 0.5$ to represent the structure. The nodes whose nodal density is 0.5 are the control points of the spline curves directly. As shown in Fig. 10, for the nodes whose nodal densities are 0.25 or 0.75, the positions of the corresponding control points are moved inward or outward from the node to the nodal density contour for $\xi = 0.5$, using simple linear interpolation. For the cantilever beam example, Fig. 11

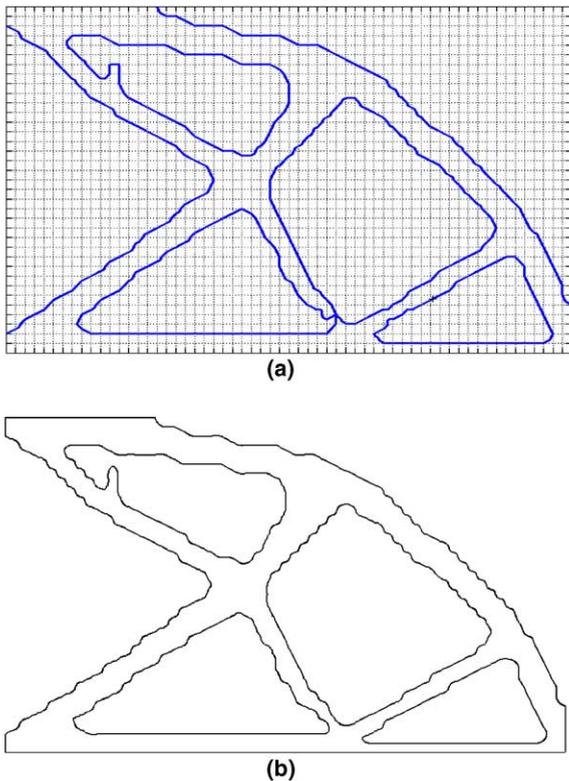


Fig. 11. Representing the topology result of the cantilever beam example using spline curves: (a) the nodal density contour and (b) the approximate nodal density contour.

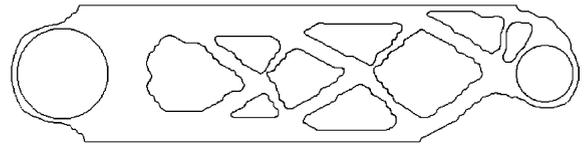


Fig. 12. Representing the topology result of the suspension arm example using spline curves.

compares the nodal density contour for $\xi = 0.5$ and the approximate nodal density contour using spline curves, which can be used in later design stages. Fig. 12 shows the resulting CAD model of the suspension arm example using the approximate nodal density contour.

5. Interpreting the three-dimensional topology optimization result

The interpretation process presented above can be directly extended to three-dimensional structures. In the interpretation process, first the same continuity analysis, trivial solid and void filter are applied as in two-dimensional interpretation process to obtain a continuous and clear three-dimensional topology optimization result.

The trivial solid for three-dimensional topology optimization result is defined as the solid containing a group of elements that does not connect with other groups, or connects with other groups in less than two nodes. Fig. 13 shows the examples for solid elements that are connected and not connected. This definition follows the criterion presented by Harzheim and Graf [7]. The definition of trivial void for the three-dimensional topology optimization result is the same as the two-dimensional topology optimization result. If the number of elements contained in a void is less than 1% of the total number of elements multiplied by $\rho_{0.0}$, the ratio of the amount of usable material, it is defined as a trivial void.

How to present a three-dimensional structure is an important issue for interpreting three-dimensional topology optimization result. Hsu et al. [8] used cross-sections to interpret the three-dimensional topology optimization

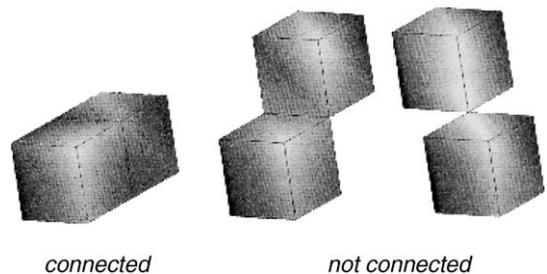


Fig. 13. Examples for solid elements, connected and not connected [7].

result. Tang and Chang [19] also used a series of pre-selected cross-sections to reconstruct the three-dimensional topology optimization result. Representative cross-sections are used to interpret the three-dimensional topology optimization result in this research.

Referring to Fig. 1, in the “representative cross-section selection operation” developed in this research, the selection of the cross-sections can be along x , y , or z direction, and the distance between cross-sections can be non-uniform. Table 3 shows the topology optimization result and boundaries of selected cross-sections of a cantilever beam example. There are seven representative cross-sections selected along the y direction in this example. In each representative cross-section, the topology result becomes a two-dimensional problem. The density redistribution algorithm and density contour approach presented above can be directly applied to obtain the boundary of the section. Any CAD software can be used to construct the three-dimensional CAD model by sweeping through the boundaries of the representative cross-sections. Fig. 14 is the reconstructed three-dimensional CAD model of the cantilever beam example.

It should be noted that the selected number of representative cross-section is related to the resolution of

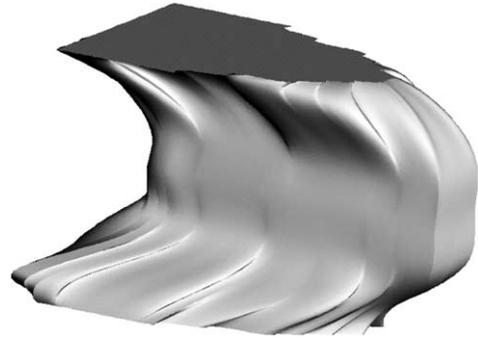


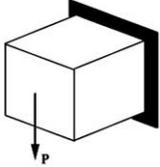
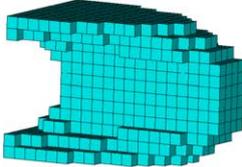
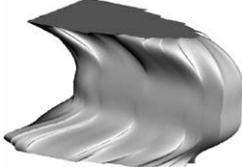
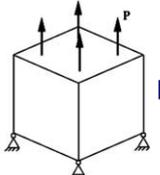
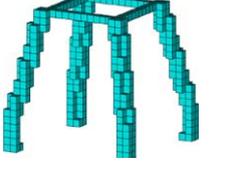
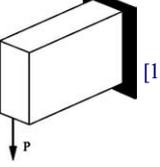
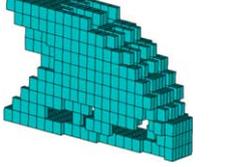
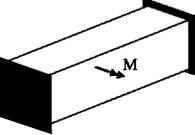
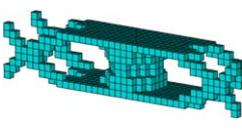
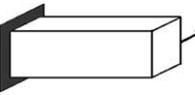
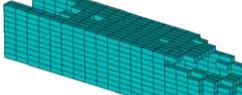
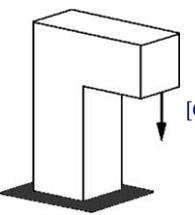
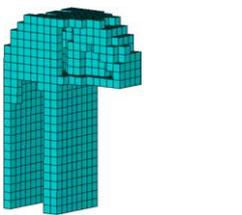
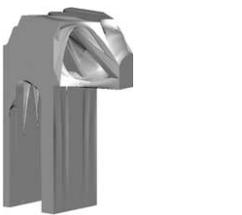
Fig. 14. The reconstructed three-dimensional CAD model of Example 1.

three-dimensional CAD model. More representative cross-sections are needed when the resolution of three-dimensional model is required. On the other hand, the direction of selected representative cross-section will affect the efficiency in reconstructing the three-dimensional CAD model. Table 4 lists 6 three-dimensional structures commonly seen in structural topology optimization literature, their topology optimization results,

Table 3
Topology results and boundaries of selected sections of a cantilever beam example (selected along y direction)

	Topology result of sections	Topology boundaries of sections		Topology result of sections	Topology boundaries of sections
1			5		
2			6		
3			7		
4					

Table 4
Topology optimization results and the reconstructed CAD models

Examples	Topology optimization results	Reconstructed CAD models
 [14,12,3]		
 [14,12]		
 [14,12]		
 [14,12,3]		
 [6]		
 [6]		

and their interpretation results using the process presented in this research.

6. Conclusion

This paper presents an automated process for interpreting three-dimensional topology optimization result from a gray level image in discrete finite elements into a smooth CAD representation. The complexity of the interpretation process depends heavily on the quality

of the topology optimization result. Therefore in this research, a tuning process is employed before the interpretation process. However, there are still limitations to the process if the topology optimization result is very poor:

- (1) The continuity analysis discussed in this paper will not work well if there is a large region full of elements with intermediate densities. Adding penalty function in the topology optimization process is a common strategy in order to get a clear topology optimization result.

- (2) The filters for trivial solids and trivial voids will not work well if the so-called “checkerboard pattern” appears in the topology optimization results. A common strategy to resolve the checkerboard problem is to use higher order finite elements.
- (3) The topology optimization model with finer finite element mesh will increase the complexity of the resulting topology, the so-called “mesh-dependence problem” [18]. Too many unnecessary details will complicate the interpretation process but not necessarily reduces the compliance of the structure. Therefore, in the topology optimization phase, we should first decide the proper number of elements in the finite element mesh.

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