



Interpreting results from topology optimization using density contours

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Abstract

The topology optimization result using material distribution method is a density distribution of the finite elements in the design domain. Interpreting this result has been a major difficulty for integrating topology optimization and shape optimization into an automated structure design procedure. It is also one of the factors that limit the extension of the optimization methods of two-dimensional structures into three-dimensional structures.

This paper presents a process for integrating topology optimization and shape optimization. In this process, density contours are used to interpret the topology optimization result, and then further integrate with shape optimization. This is a fully automated procedure, since during this process no human interpretation or intervening is required. This procedure can be extended to the topology and shape optimization of three-dimensional structures. The optimizations of two- and three-dimensional structures are presented in design examples. © 2001 Elsevier Science Ltd. All rights reserved.

Keywords: Topology optimization; Shape optimization; Density contours; Material distribution method

1. Introduction

In structural shape optimization, as in most design optimization problems, an initial design model has to be defined first. During the optimization process, the size and shape of the structure may be altered, but the topology of the structure is not changed. If the topology of the initial design model is not optimal, the result after shape optimization using this initial topology is only the “optimal design” under this non-optimal topology. Therefore topology optimization, which helps designer to find the optimal topology of the structure, has been a very active research field.

Table 1 shows the structure design examples commonly used in topology optimization literature. Three

different methods for topology optimization can be found in literature. One is the homogenization method, which is based on the assumption of a microstructure in which the properties are homogenized. Another approach is the material distribution method, in which the material density of each element is selected as the design variable. During the optimization process, intermediate density is penalized to force the design variables to approach 0 or 1. If the material density of an element is close to 0 at the end of topology optimization, the element does not exist in the optimal topology. In these two methods, the objective functions are usually to minimize compliance of the structure, or to maximize stiffness of the structure. Maximizing the lowest natural frequency of the structure is also used as the objective function in some examples. In almost all examples, the only constraint is that the amount of material that can be used in the design domain is limited. Stress constraints are usually not included in topology optimization, though Yang [1] considered stress constraints using global stress functions to approximate local stresses.

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Table 1
Structure design examples commonly used in literature

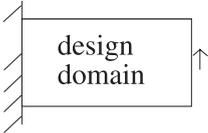
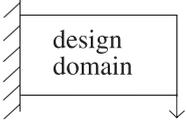
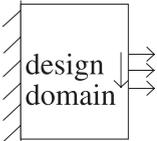
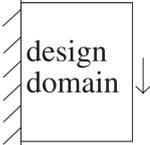
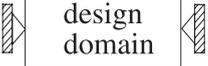
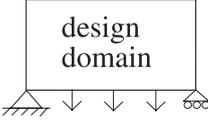
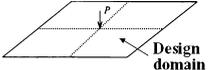
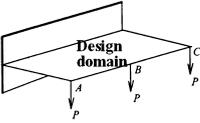
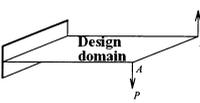
Design example	Authors	Objective	Optimization method
	a. Suzuki and Kikuhi [16] b. Bendsøe and Rodrigues [9] c. Thomsen [17] d. Ma et al. [18] e. Yang and Chuang [6] f. Xie and Steven [2] g. Chu et al. [3]	a. Compliance b. Stiffness c. Stiffness d. Compliance e. Compliance f. Compliance g. Weight	a. Homogenization b. Material distribution c. Homogenization d. Homogenization e. Material distribution f. Evolutionary method g. Evolutionary method
	a. Olhoff et al. [10] b. Mlejnek [19] c. Maute and Ramm [7]	a. Compliance b. Compliance c. Stiffness	a. Material distribution b. Homogenization c. Material distribution
	a. Bendsøe and Kikuchi [20] b. Suzuki and Kikuchi [16] c. Bendsøe and Rodrigues [9]	a. Compliance b. Compliance c. Stiffness	a. Homogenization b. Homogenization c. Homogenization
	a. Mlejnek [19] b. Mlejnek and Schirrmacher [21] c. Youn and Park [12]	a. Compliance b. Compliance c. Compliance	a. Homogenization b. Material distribution c. Homogenization
	a. Suzuki and Kikuchi [16] b. Mlejnek and Schirrmacher [21] c. Xie and Steven [2]	a. Compliance b. Compliance c. Compliance	a. Homogenization b. Material distribution c. Evolutionary method
	a. Suzuki and Kikuchi [16] b. Yang and Chuang [6] c. Chu et al. [3]	a. Compliance b. Compliance c. Weight	a. Homogenization b. Material distribution c. Evolutionary method
	a. Yang and Chuang [6] b. Ma et al. [22]	a. Natural frequency b. Natural frequency	a. Homogenization b. Material distribution
	a. Yang and Chuang [6] b. Maute and Ramm [7] c. Xie and Steven [2] d. Youn and Park [12]	a. Compliance b. Stiffness c. Compliance d. Compliance	a. Material distribution b. Material distribution c. Evolutionary method d. Homogenization
	a. Chu et al. [4,5]	a. Displacement	a. Evolutionary method

Table 1 (continued)

Design example	Authors	Objective	Optimization method
	a. Chu et al. [4]	a. Strain energy	a. Evolutionary method
	a. Chu et al. [4]	a. Strain energy	a. Evolutionary method

The third approach is the evolutionary structural optimization (ESO) first proposed by Xie and Steven [2]. The original idea of this method is to gradually remove lowly stressed elements to achieve the optimal design. Chu et al. [3] extended ESO to shape optimization problems that minimize the weight of the structure subject to stiffness constraints. Instead of completely removing the redundant elements, Chu et al. [4,5] further suggested an evolutionary method for optimal design of plates with discrete variable thickness. Plates with three-dimensional loading are used as design examples.

The first example in Table 1 is shown in more details in Fig. 1(a). Fig. 1(b) shows the optimal topology obtained by Yang and Chung [6] using the material distribution method. As shown in the figure, the result from topology optimization by any of the three methods is often a non-smooth, or even non-continuous, skeleton

type of structure, which may not be practical. Further postprocessing, either by human interpretation using certain smoothing algorithms [7], or integrating with shape optimization algorithms [8–10], is required.

When integrating with shape optimization, cubic splines are often used to ensure the smoothness of the boundary shape, and constraints on local stress are emphasized. The constraint on the total mass of the structure is often treated as a “guideline” instead of a rigid constraint, and is often relaxed in the postprocessing. Another issue that has to be resolved is the formation of the so-called “checker-board” patterns [11,12] during the topology optimization process. Youn and Park [12] suggested a density redistribution algorithm to suppress the checker-board patterns of material densities obtained from the material distribution method. Guedes and Taylor [13] presented a method in which the optimal material properties are predicted along with the “high-resolution” definition of structural shape. Through this filtering process, the originally opaque design is rendered into a distinct, “high-resolution” design.

The topology optimization result using material distribution method is a “density distribution” of the finite elements in the design domain. Kumar and Gossard [14] used contours of a “shape density function” to represent the boundaries of the shape of structural components where both shape and topology are optimized. Both shape and topology of the structure are modified simultaneously by the optimization algorithm. In their work, the shape and topology of the structure are optimized with the objective of minimizing the compliance subject to a constraint on the total mass of the structure.

This paper presents a process that uses density contours to interpret the result obtained from topology optimization, and then integrated with shape optimization. In this process, smooth boundary can be achieved, discontinuity in structure can be identified, and both the stress constraint and the constraint on total mass of the structure are considered and satisfied. The whole process is automated and can be easily implemented for computer application, which is very crucial for integrating

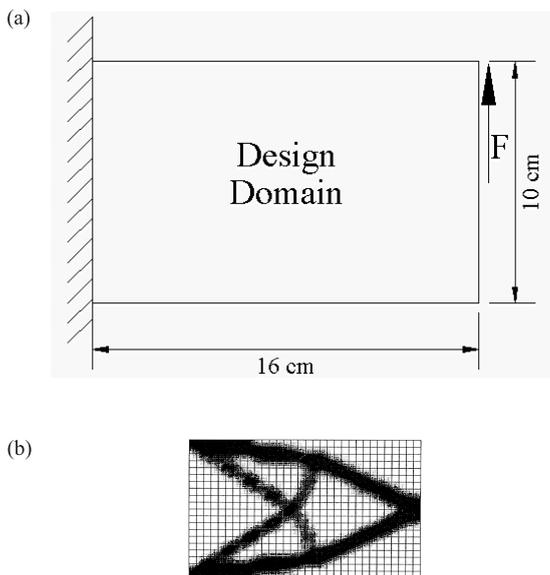


Fig. 1. The cantilever beam example: (a) the design domain and boundary conditions and (b) the result from topology optimization [6].

topology optimization with shape optimization into a fully automated structure design procedure. A two-dimensional structure is used to illustrate the ideas. However, this procedure can be easily extended to the topology and shape optimization of three-dimensional structures. Optimizations of two- and three-dimensional structures are presented in design examples.

2. The density contours

As discussed in the Section 1, the material distribution method is commonly used in topology optimization. The design variable in this method is the normalized material density of each element, which is defined as $\rho_i = \rho_{ai} / \rho_0$, where ρ_{ai} is the assumed material density of the element i ; ρ_0 is the true material density and ρ_i is the normalized material density of the element i .

The design variable ρ_i may vary between 0 and 1. Intermediate density is usually penalized to force the design variables to approach 0 or 1. If the normalized material density of an element is close to 0 at the end of topology optimization, the element does not exist in the optimal topology. On the other hand, if the normalized material density of an element is close to 1 at the end of topology optimization, the element exists in the optimal topology.

Fig. 2 shows the design domain and boundary conditions of a two-dimensional structure, which will be used to demonstrate the process using density contours to integrate topology and shape optimization. Topology optimization using the material distribution method is

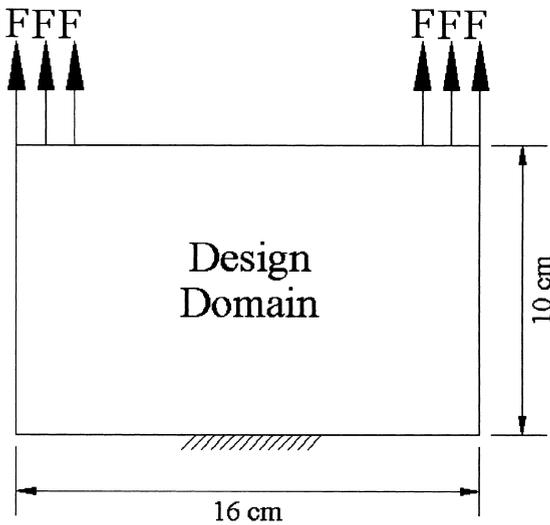


Fig. 2. The design domain of a two-dimensional structure ($F = 500$ N).

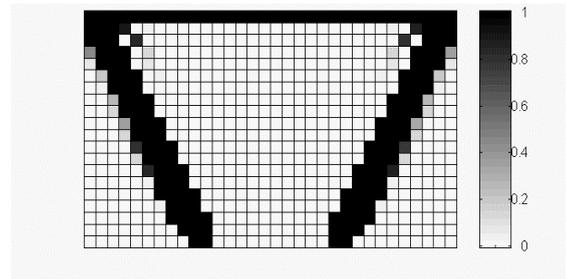


Fig. 3. The result after topology optimization.

performed on this design domain. As in most topology design cases in literature, the objective is to minimize compliance of the structure, subject to the constraint on 25% mass usage. The design domain is divided into 32×20 finite elements, which is prescribed by the user. The initial density is set at $\rho = 0.25$ for all elements in the design domain. The initial compliance of this structure is 2003.5 mmN.

Fig 3 shows the result after topology optimization. In this Figure, a black element has density value close to 1, and a blank element has density value close to 0. The compliance of the structure decreases to 315.9 mmN after 52 iterations.

The density values of the elements are then extrapolated in a non-linear fashion to obtain the nodal density values. From the nodal density values, we can easily plot the density contours for any value ρ , $0 \leq \rho \leq 1$, and compute the area covered by the contour. Fig. 4 shows the density contours for $\rho = 0.60$ and $\rho = 0.90$. The total area of the design domain in this case is 160 cm². The area for the contour for $\rho = 0.60$ is 76.0 cm², which violates the constraint on 25% mass usage. The area for the contour for $\rho = 0.90$ is 37.8 cm², which is less than the constraint on 25% mass usage. Using simple secant method, we can find that the area for the contour $\rho = 0.86$ is exactly 40.0 cm², which satisfies the constraint on 25% mass usage with strict equality.

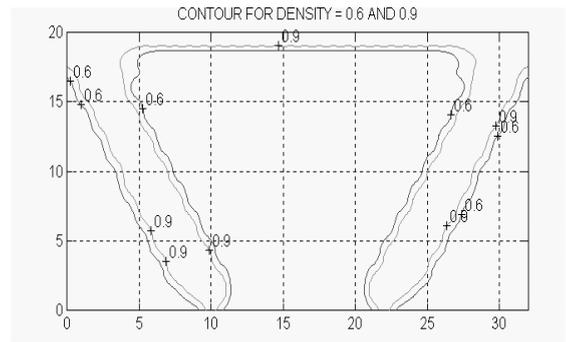


Fig. 4. Density contours for $\rho = 0.6$ and $\rho = 0.9$.

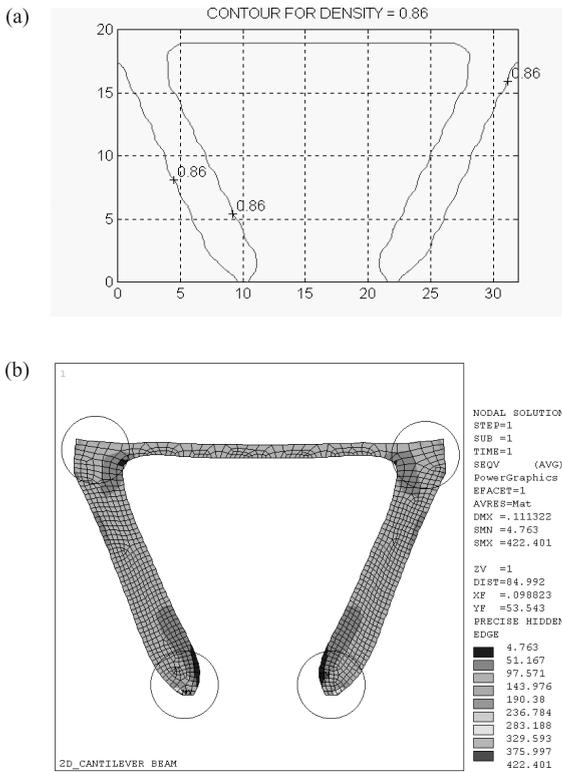


Fig. 5. The result of topology optimization: (a) density contours for $\rho = 0.86$ and (b) stress analysis results.

Fig. 5(a) shows this design. Compared with the result shown in Fig. 3, this design has a smooth and continuous boundary, and can be used as the output of topology optimization. The compliance of this design is 269.1 mmN, which is lower than that of Fig. 3. Using finite element analysis on this design, stress concentration is found around the inside corners of the structure (Fig. 5(b)). The maximum stress of this structure is 423.4 MPa, which is higher than the yielding stress of the material 280.0 MPa. Obviously further shape optimization is needed in order to satisfy the stress constraint.

3. Integrating with shape optimization

The design in Fig. 5 is used as the initial design for shape optimization. The objective is still to minimize the compliance of the structure, subject to mass and stress constraints. As shown in Fig. 6, spline curves are used to represent the initial shape. The movements along the normal vectors of the spline curves at the control points are used as design variables. As shown in the Figure, 28 equally spaced control points are automatically selected to define the boundary spline curves, so there are 28 design variables. The number of control points used in

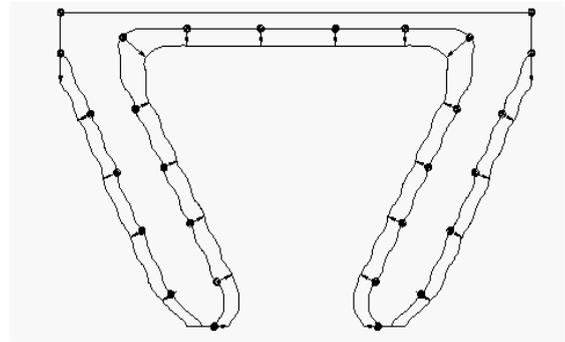


Fig. 6. Control points for the initial design model for shape optimization: (a) density contours for $\rho = 0.86$ and (b) stress analysis results.

shape optimization is also prescribed by the user. “Method of centers [15]” is used as the optimization algorithm, and the sensitivities of the design variables are calculated by finite difference.

Fig. 7 shows the shape optimization result after 40 iterations. The compliance of the structure further drops to 231.8 mmN, the area is 39.7 cm², while the maximum stress of the structure drops to 261.1 MPa.

In summary, this structure design experiences four different stages:

1. Defining the initial design domain (Fig. 2).
2. Topology optimization, to minimize compliance under the constraint on mass usage (Fig. 3).
3. Using the density values obtained in (2) to generate a smooth density contour of the structure, considering the constraint on mass usage (Fig. 5).

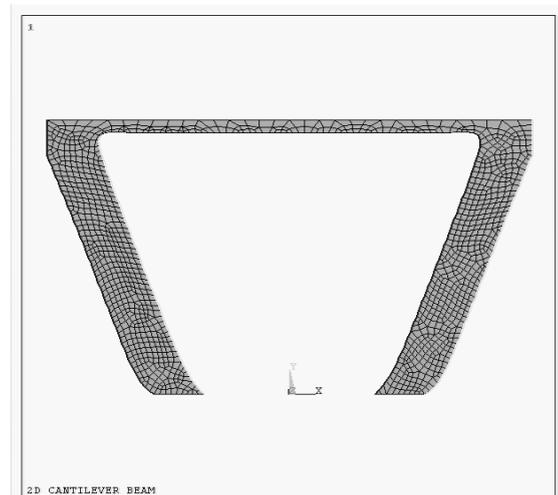


Fig. 7. The result of shape optimization.

- Shape optimization, to minimize compliance under the constraint on mass usage and stress, using the result in (3) as the initial design (Fig. 7).

This can be a fully integrated and fully automated process, since during this process no human interpretation or intervening is required. Comparing with the designs in Figs. 2 and 7, both topology and shape of the structure are optimized. The compliance of the structure drops from 2003.5 to 231.8 mm N, while the constraints on mass usage and stress are both satisfied.

4. Identifying discontinuities of the structure

The result from topology optimization may have wide spread porous regions or checker-board patterns. Discontinuity may occur when generating the design contour in stage (3) discussed above. To make the procedure described in Section 3 a fully automated process, discontinuities of the structure should also be identified automatically in order to create a reasonable structure.

Fig. 8 shows the result of the topology optimization problem defined in Fig. 1(a), using 32×10 grids. This result is obtained by the material distribution method, but no penalty function is introduced to push the density values of the elements into 0 and 1. As shown in Fig. 8, many elements have intermediate density values. Fig. 9 shows the density contours of $\rho = 0.60$. There is a discontinuity within the design domain, which has to be identified automatically before sending the density contour as the initial design for shape optimization.

In the density contour plot, a close region in the design domain is usually a “hole”. But as shown in Fig. 9, the close regions may also be discontinuous structure. Note that the boundary of the design domain is also treated as part of the close region. To identify those discontinuities for a certain density level ρ , first the close regions on the contour plot are identified, and their areas are calculated. Then as shown in Fig. 10, a slightly

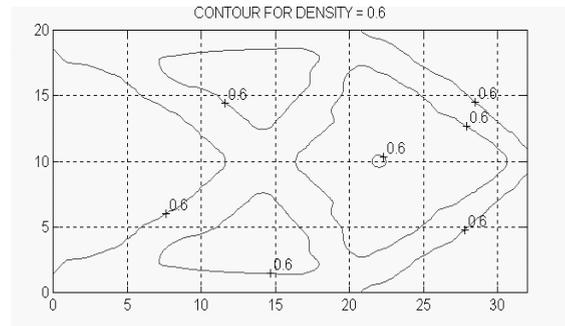


Fig. 9. Density contour for $\rho = 0.60$.

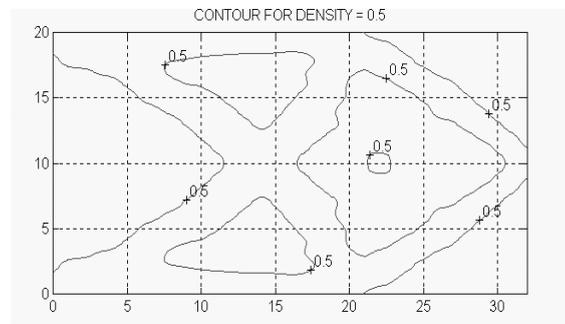


Fig. 10. Density contour for $\rho = 0.50$.

lower density level ($\rho = 0.50$ in this case) is used. The area of the close regions in this new density contour is calculated again. The close region is a discontinuity if the area becomes larger; otherwise, it is a hole.

Several things can be done when a discontinuity is identified. If the area of the discontinuity is smaller than a prescribed value, it can be ignored and deleted. Or topology optimization should be resumed using higher penalty value to push the density values to 0 or 1. If the discontinuities cannot be improved, the designer is prompt with a message about this situation and further shape optimization cannot proceed.

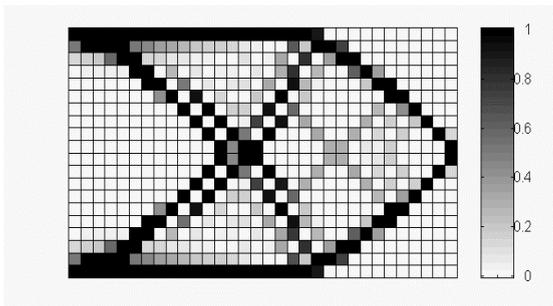


Fig. 8. The result of topology optimization of a cantilever beam.

5. Design examples

Finally three examples that are commonly used in topology optimization literatures are shown here to illustrate the practicality of this integrated procedure. For clarity, all examples are presented in the format of the four sequential stages.

5.1. The cantilever beam example

Stage 1. Defining the initial design domain: The same cantilever beam example in Fig. 1 is shown again in Fig. 11.

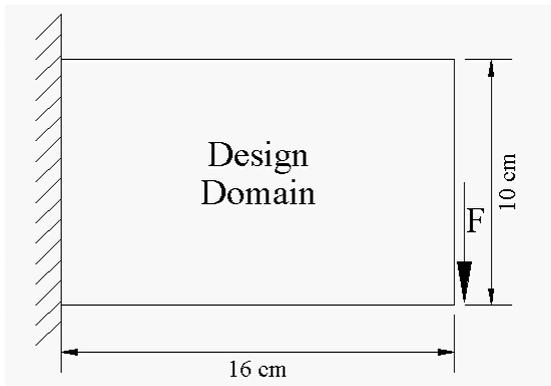


Fig. 11. The design domain of a cantilever beam ($F = 500$ N).

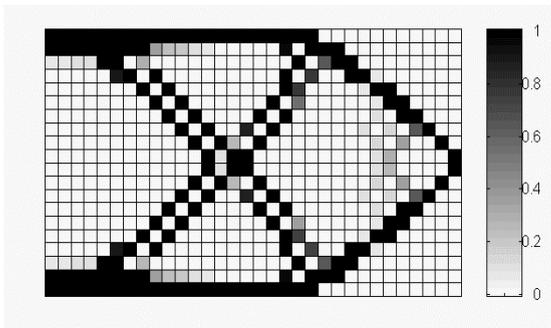


Fig. 12. Topology optimization result of the cantilever beam.

Stage 2. Topology optimization: In topology optimization, the objective is to minimize compliance of the structure, subject to the constraint on 25% mass usage. The design domain is divided into 32×20 finite elements. The initial density is set at $\rho = 0.25$ for all elements in the design domain. The initial compliance is 453.5 mmN.

Fig. 12 shows the topology optimization result after 50 iterations. Its compliance decreases to 123.7 mmN.

Stage 3. Generating the density contour: As shown in Fig. 13, the area covered by the density contour $\rho = 0.875$, is found to be exactly 40.0 cm^2 (25% of the area of the design domain). It is used as the initial structure for shape optimization. The maximum stress of this initial design is found to be 427.7 MPa, which is higher than the yielding stress of the material 280.0 MPa.

Stage 4. Shape optimization: In shape optimization, 36 control points are automatically selected from Fig. 13 as design variables. The objective is still to minimize the compliance of the structure, subject to mass and stress constraints. Fig. 14 shows the shape optimization result after 40 iterations. The compliance further decreases from 113.8 mmN (Fig. 14) to 93.8 mmN (Fig. 14). Its

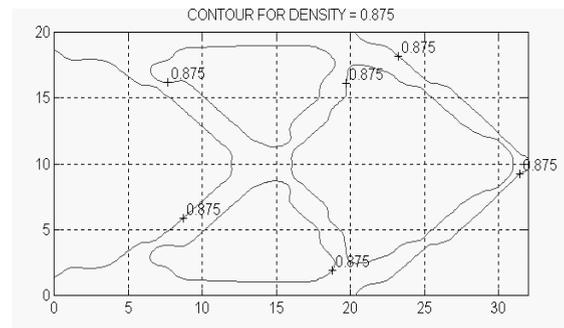


Fig. 13. Density contour for $\rho = 0.875$.

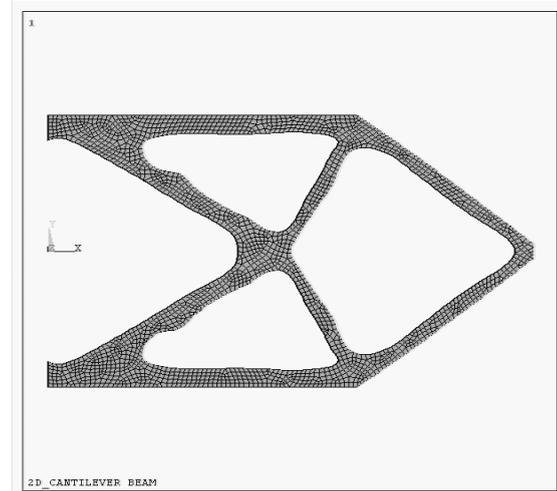


Fig. 14. Shape optimization result of the cantilever beam.

area is 39.7 cm^2 , and maximum stress is 270.6 MPa, both are lower than the constrained values.

5.2. The simply supported beam example

Stage 1. Defining the initial design domain: The dimensions, boundary conditions and loads of a simply supported beam are shown in Fig. 15.

Stage 2. Topology optimization: In topology optimization, the objective is to minimize compliance of the structure, subject to the constraint on 25% mass usage. The design domain is divided into 64×40 finite elements. The initial density is set at $\rho = 0.25$ for all elements in the design domain. The initial compliance is 1162.8 mmN. Fig. 16 shows the topology optimization result after 50 iterations. Its compliance decreases to 159.7 mmN.

Stage 3. Generating the density contour: As shown in Fig. 17, the area covered by the density contour $\rho = 0.9$, is found to be exactly 40.0 cm^2 (25% of the area of the

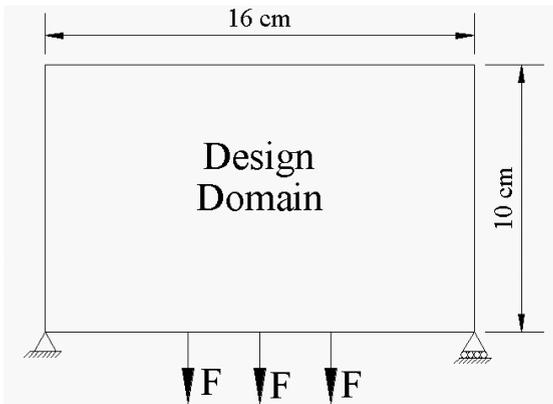


Fig. 15. The design domain of a simply supported beam ($F = 500 \text{ N}$).

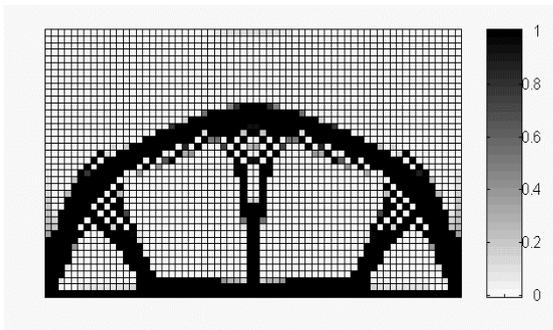


Fig. 16. Topology optimization result of the simply supported beam.

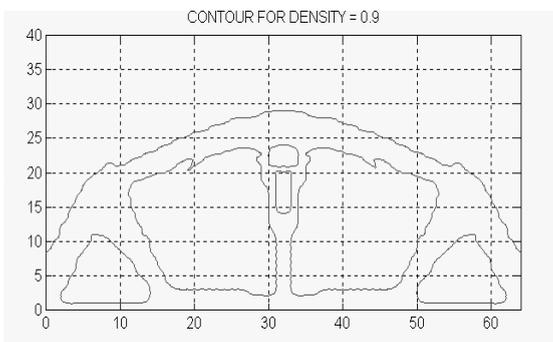


Fig. 17. Density contour for $\rho = 0.9$.

design domain). It is used as the initial structure for shape optimization.

Stage 4. Shape optimization: For shape optimization, 36 control points are automatically selected from Fig. 17 as design variables. The objective is still to minimize the compliance of the structure, subject to mass and stress

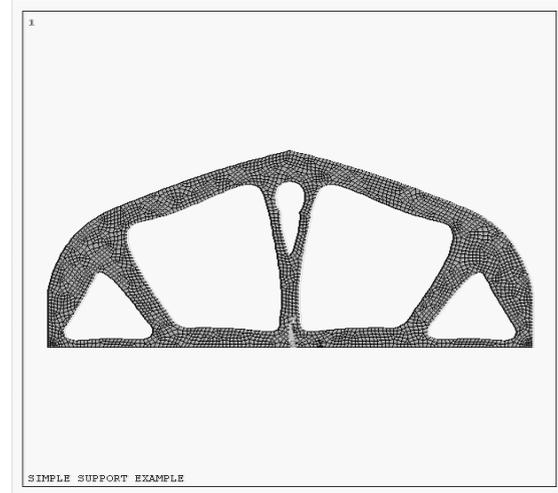


Fig. 18. Shape optimization result of the simply supported beam.

constraints. Fig. 18 shows the shape optimization result after 40 iterations. The compliance further decreases from 162.0 mmN (Fig. 17) to 160.8 mmN (Fig. 18). Its area is 39.7 cm², and maximum stress is 80.0 MPa, both are lower than the constrained values.

5.3. The three-dimensional cantilever beam

The following example is a three-dimensional cantilever beam that is widely used in literature discussing three-dimensional structural topology and shape optimization [2,23–25]. It is presented here to illustrate that all four stages of the procedure developed in this paper can be easily extended to three-dimensional structural optimization.

Stage 1. Defining the initial design domain: The three-dimensional design domain, boundary conditions and load of a three-dimensional cantilever beam are shown in Fig. 19.

Stage 2. Topology optimization: In topology optimization, the objective is to minimize compliance of the structure, subject to the constraint on 25% mass usage. The three-dimensional design domain is divided into $8 \times 4 \times 4$ finite elements. The initial density is set at $\rho = 0.25$ for all elements in the design domain. The initial compliance is 52,485 mmN. Fig. 20 shows the topology optimization result after 75 iterations. Its compliance decreases to 11,516.5 mmN.

Stage 3. Generating the density contour: As shown in Fig. 21, the three-dimensional design domain is represented by 9 cross-sections. Each cross-section is a two-dimensional plane, and the same method described in the previous sections can be used to find the density contours for each cross-section. 86 control points are

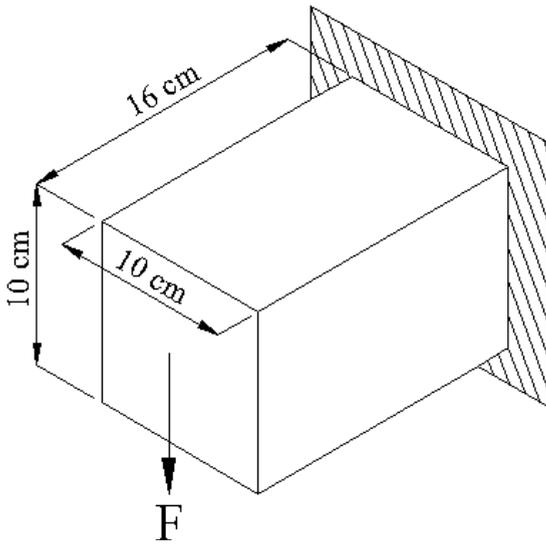


Fig. 19. The design domain of a three-dimensional cantilever beam ($F = 50000$ N).

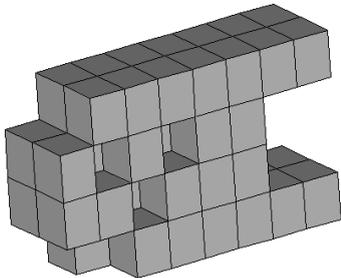


Fig. 20. Topology optimization result of a three-dimensional cantilever beam.

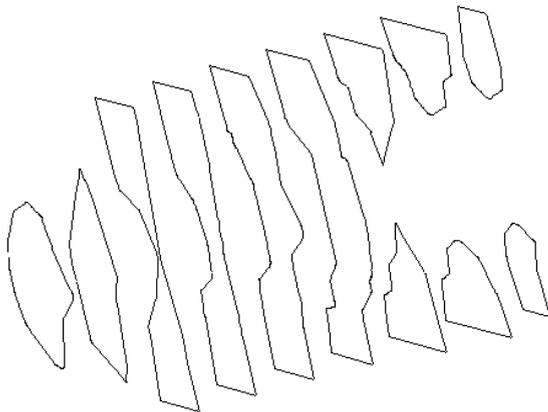


Fig. 21. Density contour for $\rho = 0.75$ of each cross-section.

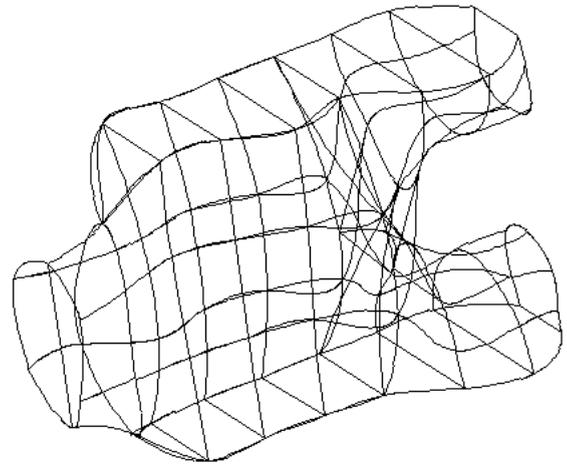


Fig. 22. The volume of the initial design of the three-dimensional cantilever beam.

automatically selected from Fig. 21 as design variables. Spline curves are used to connect control points of each cross-section. The density contours of each cross-section are then swept into a three-dimensional volume, as shown in Fig. 22. The volume covered by the density contour $\rho = 0.75$, is found to be 412.48 cm^3 (about 25% of the volume of the design domain). It is used as the initial structure for shape optimization. The maximum stress of this initial design is found to be 357.34 MPa , which is higher than the yielding stress of the material 280.0 MPa .

Stage 4. Shape optimization: For shape optimization, the objective is still to minimize the compliance of the structure, subject to mass and stress constraints. Fig. 23 shows the shape optimization result after 25 iterations. The compliance further decreases from $11,391.86 \text{ mm N}$ (Fig. 22) to $11,339 \text{ mm N}$ (Fig. 23). Its volume is 383.17 cm^3 , and maximum stress is 272.7 MPa , both are lower than the constrained values.

6. Discussions and conclusions

As described in previous sections, the topology optimization result using material distribution method is a density distribution of the finite elements in the design domain. Interpreting this result has always been a major difficulty for integrating topology optimization and shape optimization into an automated structure design procedure.

This paper presents a procedure for integrating topology optimization and shape optimization. In this procedure, density contours are used to interpret the topology optimization result, and then further integrate with shape optimization. As illustrated in the design

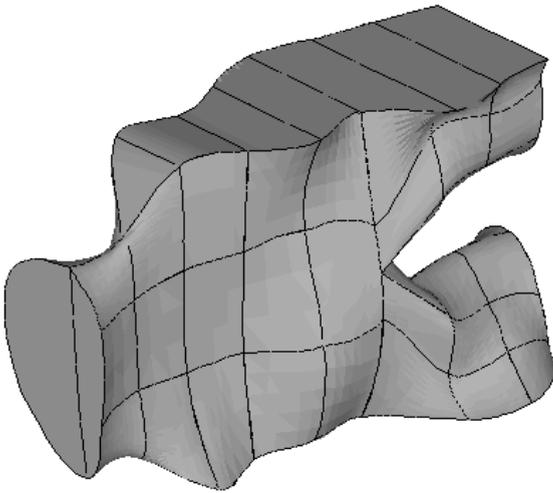


Fig. 23. The shape optimization result of the three-dimensional cantilever beam.

examples, this is a fully automated procedure, since during this process no human interpretation or intervening is required. In this process, smooth boundary can be achieved, discontinuity in structure can be identified, and both constraints on stress and mass usage are satisfied.

The difficulty of interpreting the result obtained from topology optimization has also limited the extension of the methods of two-dimensional structure optimal design into three-dimensional structures. All four stages in the procedure developed in this paper can be easily extended to three-dimensional structures. The topology and shape optimization of a three-dimensional structure using this procedure is also presented.

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