

# Fully stressed thickness profile design at discontinuities of axisymmetric shells

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**Abstract** In a shell structure, the discontinuity at the intersection of two shells causes stress concentration. This paper presents a procedure which couples the Curvature Function Method with the FAST1 structural shell analysis program to find a fully stressed thickness profile which keeps the stress at the discontinuities at the nominal stress value. The Curvature Function Method is a zero-order method that requires only stress values along the shell, not gradients of the stresses with respect to the design variables, and the resulting thickness profile has  $C^2$  continuity. Although the method is independent of the structural analysis program used to determine stress values, Fast1 provides a particular advantage because it allows the user to model complex shells with only a few large shell elements and still retain a sufficiently accurate solution. Thus both preparation and computation times are reduced substantially. Convergence of different initial designs to one final design using this procedure is demonstrated for a cylinder-cone intersection problem. This procedure is also applied to two other shell models with multiple discontinuities to find their fully stressed thickness profiles. The procedure presented in this paper provides a practical technique and tool to aid the design engineer, although the fully stressed design may not be the theoretically optimal design of minimum weight.

## 1 Introduction

In a shell structure, the discontinuity at the intersection of two shells causes stress concentration. The magnitude of the stress depends on the magnitude of the discontinuity. If a discontinuity in the slope of the meridian occurs, the stress concentration will be of the order of magnitude of the square root of radius-to-thickness. This is referred to as the Type I discontinuity. If the slope is continuous but the radius of the meridian is discontinuous, the stress concentration is of an order of 1. This is referred to as the Type II discontinuity (Steele and Balch 1990).

Stress concentration at the discontinuities often causes failure of the shell structure; thus how to reinforce the discontinuities to minimize stress concentration while keeping the weight of the shell structure at the lowest possible value is an important task in designing shell structures. This paper presents a procedure which couples the Curvature Function Method with the FAST1 shell analysis program to find a fully stressed thickness profile which keeps the stress at the discontinuities at the nominal stress value.

The Curvature Function Method is a zero-order method; only one structural analysis is needed for each iteration, and no gradient calculation is required. The curvatures of the thickness curve are used as design variables. Since it appears that the local curvature of the thickness profile has a monotonic relation with stress, a zero-order search direction can be defined to search for the curvature function which achieves a fully stressed boundary. There is no restriction on the number of variables used to represent the curve because the computational effort is independent of the number of variables, and the resulting thickness curve integrated from the curvatures will have  $C^2$  continuity.

The FAST1 program is used for the analysis of a general axisymmetric shell structure. It allows the user to model complex shells with only a few large shell elements and still retain a sufficiently accurate solution; thus both preparation and computation times are reduced substantially in comparison with the many finite element programs available. The Curvature Function Method is independent of the structural analysis program. It is demonstrated in this paper how the FAST1 program can be easily coupled with the Curvature Function Method without modifying the source code of the FAST1 program.

A cylinder-cone intersection problem is used to demonstrate the ease of preparation of a shell model for this fully stressed thickness profile design procedure. Convergence of different initial designs to one final design using this procedure is also demonstrated for the cylinder-cone intersection problem. Two other shell thickness redesign examples, which have multiple discontinuities, are presented.

Fully stressed design is commonly used in structural optimization (e.g. Lansing *et al.* 1971). One may argue that for statically indeterminate structures, the theoretically optimal design of minimum weight may not be fully stressed (Razani 1965; Dayaratram and Patnik 1969). However, in structural shape optimization, global optimality was never proved in any algorithm, and even local optimality could not be proved because of the natural restriction in the geometrical representation of boundary shape. Moreover, in most cases of a structure made of a single material, there is a fully stressed design near the optimum design (Haftka *et al.* 1990). So practically speaking, the fully stressed thickness design

for a shell structure is certainly useful and desirable. To the authors' knowledge, papers published on the optimization of thin shell structures are still rather restricted, and no practical technique and tool that can efficiently aid a design engineer has been presented. In this regard, the procedure and computer program presented in this paper represent a substantial advance.

## 2 The curvature function method

### 2.1 Finding the curvature function for a fully stressed design

Most research in shape optimization is focussed on interfacing structural analysis programs directly with first-order optimization algorithms, e.g. sequential linear programming (Zienkiewicz and Campbell 1973; Pedersen and Laursen 1982-83) and the CONMIN feasible direction algorithm (Vanderplaats 1973; Botkin 1982). Jacoby (1992) also combined the GENOPT optimization development system (Bushnell 1990) with the FAST1 shell analysis program for shell thickness and shape optimization. These algorithms require the gradient of constraints on structural responses such as stresses, displacements and natural frequencies. Calculating the gradients by finite difference is computationally expensive when the number of design variables is large. On the other hand, analytical methods for gradient calculation are computationally more efficient, but considerable coding is required by experts in the field to implement them, and the available design variables are still limited.

The difficulties in obtaining first-order information motivated the development of the Curvature Function Method (Hsu 1992, pp. 24-29). This is a zero-order method. Only one structural analysis is needed for each iteration, because no gradient calculation is required. The curvatures of the thickness curve are used as design variables. To keep the stress constant at nominal stress along the boundary is used as the design criteria.

Figure 1 shows two thickness profiles of a shell segment. It is assumed that the thickness profiles are symmetric with respect to the mid surface. The stress value at a point on the profile is determined by the nominal stress and the stress concentration effect. Nominal stress depends on the load and the amount of material carrying the load. Obviously, the nominal stress can be reduced by increasing the thickness and vice versa. Stress concentration depends on the smoothness of local geometry. An abrupt change in thickness results in high stress concentration.

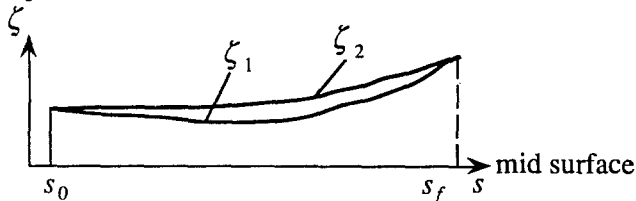


Fig. 1. Thickness curves

Boundary smoothness has been one of the key factors for the success of shape optimization problems under stress constraints. For example, researchers have used spline curves, in

which smoothness is a built-in feature, for boundary representation. However, as far as the authors are aware, there has not yet been an explicit measure of smoothness used in the optimization process. The curvature of the boundary curve can be used for this purpose since it is closely related to stress concentration effects. Peterson (1974) demonstrated the well-known fact that the stress concentration factor at a notch or a shoulder fillet increases monotonically as the radius (inverse of curvature) of the notch or fillet decreases. The basis for Peterson's results were experiments. Recent research by Gao (1991) gives an analytical solution for stress concentration at slightly undulating surfaces, which shows the same monotonic relation between the curvature of the boundary curve and the stress concentration effect.

The most intuitive way to achieve a fully stressed boundary is to increase the shell thickness at the region where the stress is high and reduce the thickness if the stress is low. However, simply increasing thickness does not ensure a decrease in stress since this may create sharp corners which cause stress concentration. On the other hand, the theorem below, which was proved by Hsu (1992, pp. 24-29), states that when the positions of both end points of the thickness curve to be varied are fixed, decreasing curvature of the thickness curve will also increase the thickness, which ensures a decrease in stress.

**Theorem.** Refer to Fig. 1, let  $\zeta_1(s)$  and  $\zeta_2(s)$  be continuous thickness curves for  $s_0 \leq s \leq s_f$ , with curvature functions  $\kappa_1(s)$  and  $\kappa_2(s)$ , respectively. Let  $\zeta_1(s_0) = \zeta_2(s_0)$ ,  $\zeta_1(s_f) = \zeta_2(s_f)$ . If  $\kappa_1(s) \leq \kappa_2(s)$  for all  $s_0 \leq s \leq s_f$ , then  $\zeta_1(s) \geq \zeta_2(s)$  for all  $s_0 \leq s \leq s_f$ .

With this strictly monotonic relationship between the curvature of the thickness curve and stress (not just the stress concentration effect), it is much more convenient to work in the curvature domain when trying to achieve the fully stressed boundary. The curvature is increased if the stress is lower than the target stress, and decreased if the stress is higher than the target stress.

The curvature function of classical analytical geometry in the  $\zeta$ - $s$  plane is (Mortenson 1985, p. 273),

$$\kappa(s) = \frac{d^2\zeta/ds^2}{[1 + (d\zeta/ds)^2]^{3/2}}, \quad (1)$$

which cannot be solved analytically. A discrete version of (1) is used below to develop an algorithm applicable to digital computers. The thickness curve is discretized into  $n$  subsets, and its curvature function is represented by a corresponding piecewise linear function. Curvatures  $\kappa_i$  at the  $i$ -th point,  $i = 1, \dots, n-1$ , are the control (decision) variables, the thickness  $\zeta_i$  at the  $i$ -th point becomes the state (solution) variables. The task is to find a set of  $\kappa_i$  for which the resulting design satisfies the stress constraints with strict equality. Simple numerical integration can be used to determine  $\zeta(s)$ , given  $\kappa(s)$  and  $\zeta(s_0)$ ,  $\zeta(s_f)$ . Note that from (1), the thickness curve  $\zeta(s)$  integrated from  $\kappa(s)$  will have  $C^2$  continuity if  $\kappa(s)$  is continuous.

### 2.2 Zero-order search direction

From the discussion above, if the stress  $\sigma_i$  at a boundary point is too high (low), then the corresponding  $\kappa_i$  is reduced (increased) in the next iteration, until the stress distribution